**Bayes – practice problems**

**Exercise 1:**

A doctor is called to see a sick child. The doctor has prior information that  
90% of sick children in that neighborhood have the flu, while the other 10% are sick with

measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that F ∪ M = Ω, i.e., that there no other maladies in that neighborhood.  
A well-known symptom of measles is a rash (the event of having which we denote R).  
Assume that the probability of having a rash if one has measles is P (R | M ) = 0.95.  
However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is P (R | F ) = 0.08.  
Upon examining the child, the doctor finds a rash. What is the probability that the child  
has measles?

P(M | R) = (0.95) \* (0.10) / (0.95 \* 0.10 + 0.80 \* 0.90) = 57%

**Exercise 2:**

In a study, physicians were asked what the odds of breast cancer would be in a woman who was initially thought to have a 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumors and 90% of benign tumors.)  
95 out of a hundred physicians estimated the probability of cancer to be about 75%. Do  
you agree?

P(Malignant | Mammogram) = P(Mam | Mal) \* P(Mal) / P(Mam | Mal) \* P(Mal) + P(Mam | Ben) \* P(Ben) = (0.80 \* 0.01) / (0.80 \* 0.01 + 0.10 \* 0.99) = 0.075. 7.5% != 75%

**Exercise 3:**

Suppose we have 3 cards identical in form except that both sides of the first  
card are colored red, both sides of the second card are colored black, and one side of the. third card is colored red and the other side is colored black.  
The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the  
ground. If the upper side of the chosen card is colored red, what is the probability that the. other side is colored black?

P(RedBlack | Red) = P(RedBlack) \* P(Red | RedBlack) / P(RedRed) \* (Red | RedRed) + P(BlackBlack) \* P(Red | BlackBlack) + P(RedBlack) \* P(Red | RedBlack)

= 0.33 \* 0.5 / 0.33 \* 1 + 0.33 \* 0 + 0.33 \* 0.5 = 0.33 = 1/3

**Exercise 4:**

It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%.  
Now if an email is detected as spam, then what is the probability that it is in fact a  
non-spam email?  
 P(Spam’ | Detected) = P(Detected | Spam’) \* P(Spam’) / P(Detected | Spam) \* P(Spam) + P(Detected | Spam’) \* P(Spam’)

= 0.05 \* 0.5 / 0.99 \* 0.5 + 0.05 \* 0.5 = 5/104